Notes on 2.1

Bisection Method:

Algorithm to find zero of a continuous function in a bounded interval where on the end points the function has different signs.

$f \in C([a, b])$, $\operatorname{sgn}(f(a)) \neq \operatorname{sgn}(f(b))$

Algorithm.

1. let $a_1 = a, b_1 = b, p_1 = a_1 + \frac{b_1 - a_1}{2}$
2. while $f(p_i) \neq 0$
   a. if $\operatorname{sgn}(f(a_i)) = \operatorname{sgn}(f(p_i))$, let $a_i = p_i$
   b. else let $b_i = p_i$
   c. repeat

note: should really stop if we are close to zero.

theorem 2.1

$f \in C([a, b])$, $\operatorname{sgn}(f(a)) \neq \operatorname{sgn}(\mathrm{f(b)})$*, then the* p_i generated from the bisection method is such that:

$$|p_n - p| \leq \frac{b-a}{2^n}$$

where p is some zero of f in (a, b)

(therefore $p_n = p + \mathcal{O}(2^{-n})$)

Proof:

1. $p_n \in (a_n, b_n)$ and $p \in (a_n, b_n)$ therefore $|p - p_n| \leq |b_n - a_n|$
2. $|b_n - a_n| = \frac{1}{2^{n-1}}(b-a)$